Implementation of distributed consensus on an outdoor testbed

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Quadrotor dynamics
Frames of reference
Thrust and torque generation
Control laws
Inner loops
Waypoint navigation
Consensus law
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Hardware
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Consensus video
Position plots
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Conclusion

Overview

Consensus: all agents reach an autonomously decided common state

Distributed: all agents take control decisions themselves

> Contribution: Distributed multiagent consensus algorithm

Convergence: theoretically provable

Practical implementation: outdoor, on three quadrotors

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Motivation

- Quadrotors: good benchmark for testing advanced control algorithms
  - Ease of assembly, availability of components
  - Nonlinear underactuated dynamics
  - Six degrees of freedom
- State of the art:
  - Sophisticated theory for multiagent consensus (single and double integrator agents) already developed
  - Practical implementation: Indoors with motion capture cameras or with centralized control algorithms
  - COLLMOT group, Eötvös University, Hungary: distributed, empirical consensus law without proof of convergence

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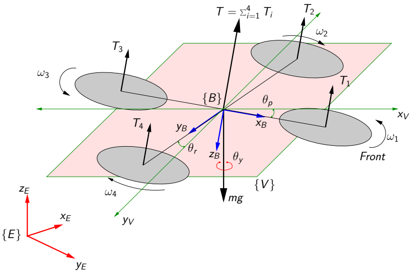
Quadrotor dynamics

Frames of reference:



Earth frame, $\{E\}$

- x_E- axis points in the north direction
- y_E- axis points in the east direction
- z_E- axis points in the up direction

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Quadrotor dynamics

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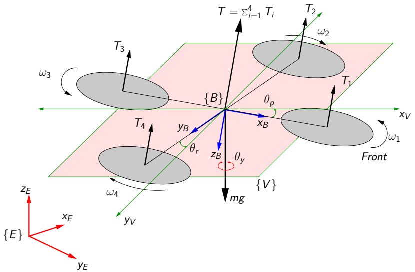

Body frame, $\{B\}$

- x_B- axis points towards front end
- y_B- axis points towards right end
- z_B- axis points downwards

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Quadrotor dynamics

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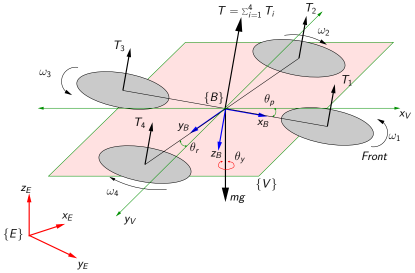

An auxiliary frame, {V},

- same origin as $\{B\}$
- z_V axis is parallel to z_E- axis
- x_V, y_V- axes are projections of x_B, y_B onto a plane parallel to the $x_E y_E-$ plane in $\{E\}$ and passing through the origin of $\{V\}$.

Quadrotor dynamics

- Motion along six degrees of freedom achieved by varying rotor speeds, $\bar{\omega}_i$
- Generating pairwise difference in rotor thrusts leads to rotational motion
-

$$\begin{bmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} -b & -b & -b & -b \\ 0 & -db & 0 & db \\ db & 0 & -db & 0 \\ k & -k & k & -k \end{bmatrix} \begin{bmatrix} \bar{\omega}_1^2 \\ \bar{\omega}_2^2 \\ \bar{\omega}_3^2 \\ \bar{\omega}_4^2 \end{bmatrix} = A \begin{bmatrix} \bar{\omega}_1^2 \\ \bar{\omega}_2^2 \\ \bar{\omega}_3^2 \\ \bar{\omega}_4^2 \end{bmatrix} \quad (1)$$

where,

  - T is thrust generated, $\begin{bmatrix} \tau_x & \tau_y & \tau_z \end{bmatrix}^T$ are the torques generated
  - b,k: constants
  - d: distance of the motor from the CoG of the quadrotor

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Control loops

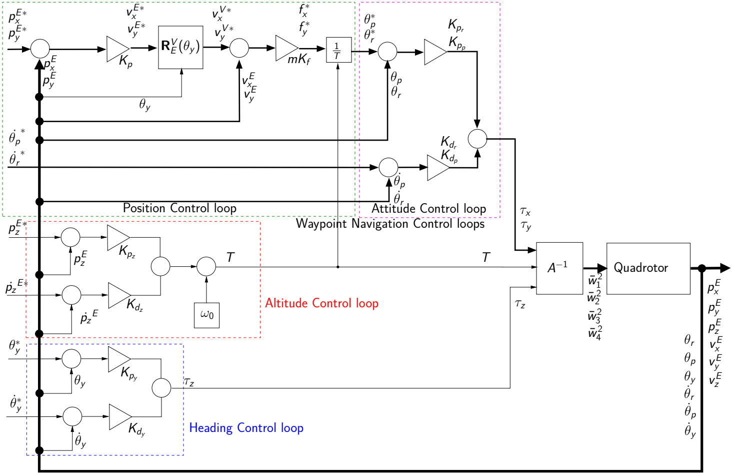

Figure: A block diagram of the quadrotor control loops

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Waypoint navigation

- Process by which the quadrotor navigates to different positions $\mathbf{p}^E = \begin{bmatrix} p_x^E & p_y^E \end{bmatrix}^T \in \mathbb{R}^2$ in $\{E\}$
- Generate τ_x and τ_y to vary θ_p and θ_r and thus maneuver the quadrotor
- Keep θ_y constant using heading control loop.

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Waypoint navigation

- Consider the frame $\{V\}$. To accelerate along x_V- and y_V- axes, we need to generate forces

$$f_x^V = T\sin\theta_p \approx T\theta_p \quad (2)$$
$$f_y^V = T\sin\theta_r\cos\theta_p \approx T\theta_r. \quad (3)$$

for small θ_x and θ_y

- We control the motion using a PD law

$$\mathbf{f}^{V*} = mK_f[K_p(\mathbf{p}^{V*} - \mathbf{p}^V) - \mathbf{v}^V] \quad (4)$$

where $\mathbf{f}^V = \begin{bmatrix} f_x^V & f_y^V \end{bmatrix}^T$

- Then, desired values of angles, $\mathbf{\Theta}^* = \begin{bmatrix} \theta_p^* & \theta_r^* \end{bmatrix} \in \mathbb{R}^2$ are

$$\mathbf{\Theta}^* = \frac{mK_f}{T}[K_p(\mathbf{p}^{E*} - \mathbf{p}^E) - \mathbf{v}^E] \quad (5)$$

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- To attain $\boldsymbol{\Theta}^* = \left[\begin{array}{cc} \theta_p^* & \theta_r^* \end{array} \right] \in \mathbb{R}^2$, generate torques $\tilde{\boldsymbol{\Gamma}} = \left[\begin{array}{cc} \tau_x & \tau_y \end{array} \right]^T \in \mathbb{R}^2$ using a PD controller

$$\tilde{\boldsymbol{\Gamma}} = K_{p_{r,p}}(\boldsymbol{\Theta}^* - \boldsymbol{\Theta}) + K_{d_{r,p}}(\dot{\boldsymbol{\Theta}}^* - \dot{\boldsymbol{\Theta}}) \tag{6}$$

  where $K_{p_{r,p}} = \left[\begin{array}{cc} K_{p_r} & K_{p_p} \end{array} \right]^T \in \mathbb{R}^2$ and $K_{d_{r,p}} = \left[\begin{array}{cc} K_{d_r} & K_{d_p} \end{array} \right]^T \in \mathbb{R}^2$ are the control gains.
- Controller designed such that $\theta_p \to \theta_p^*$ and $\theta_r \to \theta_r^*$ almost immediately

Waypoint navigation

- In the $\{E\}$ frame,

$$\mathbf{f}^E = \mathbf{R}_V^E \mathbf{f}^V \tag{7}$$

and

$$f_x^V = T \sin\theta_p \approx T\theta_p \tag{8}$$

$$f_y^V = T \sin\theta_r \cos\theta_p \approx T\theta_r. \tag{9}$$

for small θ_x and θ_y

- If we can vary θ_p and θ_r independently and instantaneously, then motion in the $x_E y_E -$ plane can be modelled as a double integrator.

Waypoint navigation

- We vary θ_p and θ_r independently and quickly such that change in angle is much faster than translational motion
- As θ_p and θ_r change, the vertical component of T reduces by a factor of the cosine of θ_p and θ_r
- But θ_p and θ_r are small and altitude control loop is fast

Hence the quadrotor can be modelled as a double integrator

$$\dot{\mathbf{p}}^E = \mathbf{v}^E, \qquad \dot{\mathbf{v}}^E = \mathbf{f}^E \tag{10}$$

where

- $\mathbf{p}^E = \begin{bmatrix} p_x^E & p_y^E \end{bmatrix}^T \in \mathbb{R}^2$ is the position in $\{E\}$
- $\mathbf{v}^E = \begin{bmatrix} v_x^E & v_y^E \end{bmatrix}^T \in \mathbb{R}^2$ is the velocity in $\{E\}$
- $\mathbf{f}^E = \begin{bmatrix} f_x^E & f_y^E \end{bmatrix}^T \in \mathbb{R}^2$ is the acceleration input in $\{E\}$

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Consensus law

- If, for all $\mathbf{p}_i^E(0)$ and $\mathbf{v}_i^E(0)$ and all $i, j = 1, ..., n$, $\|\mathbf{p}_i^E(t) - \mathbf{p}_j^E(t)\| \to 0$ and $\mathbf{v}_i^E \to 0$ as $t \to \infty$ then consensus achieved
- Information exchange modelled as undirected graph $\mathcal{G}_n := (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, ..., n\}$ is the set of nodes and $\mathcal{E} \subseteq (\mathcal{V} \times \mathcal{V})$ is the set of edges
- Node $\equiv$ quadrotor, edge $\equiv$ available communication channel
- Set of neighbours, $\mathcal{N}_i := \{j \in \mathcal{V} : (i, j) \in \mathcal{E}\}$.
- Laplacian matrix $\mathcal{L}_n$ of a graph $\mathcal{G}_n$ is given by $\mathcal{L}_n = [l_{ij}] \in \mathbb{R}^{n \times n}$; $l_{ij} = -a_{ij}, i \neq j, l_{ii} = \sum_{j=1, j \neq i}^{n} a_{ij}$.

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Consensus law

We propose the following consensus law for the n quadrotors, where each quadrotor is modelled as a double integrator

$$\mathbf{f}_i^E = \sum_{j \in \mathcal{N}_i} a_{ij}(\mathbf{p}_j^E - \mathbf{p}_i^E) - \beta \mathbf{v}_i^E, \qquad i = 1, \ldots, n \qquad (11)$$

where

- a_{ij} is the (i, j) entry of the adjacency matrix $\mathcal{A}_n \in \mathbb{R}^{n \times n}$ corresponding to the communication graph, $\mathcal{G}_n$
- β is a positive constant

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Theorem

Given a system

$$\dot{\mathbf{p}}^E = \mathbf{v}^E, \qquad \dot{\mathbf{v}}^E = \mathbf{f}^E \tag{12}$$

The control law [1],

$$\mathbf{f}_i^E = \sum_{j \in \mathcal{N}_i} a_{ij}(\mathbf{p}_j^E - \mathbf{p}_i^E) - \beta \mathbf{v}_i^E, \qquad i = 1, \ldots, n$$

achieves consensus asymptotically iff $\mathcal{G}_n$ is connected

As a result

- $\mathbf{p}(t) \to (\beta \mathbf{1}_n \mathbf{1}_n^T \otimes I_2)\mathbf{p}(0) + (\mathbf{1}_n \mathbf{1}_n^T \otimes I_2)\mathbf{v}(0)$
- $\mathbf{v}(t) \to 0$ as $t \to \infty$

Hence $\|\mathbf{p}_i^E(t) - \mathbf{p}_j^E(t)\| \to 0$ and $\mathbf{v}_i^E \to 0$ as $t \to \infty$ for all $i, j = 1, \ldots, n$

[1] Proof similar to W. Ren, R. Beard, Distributed consensus in multi-vehicle cooperative control, Springer, 2008

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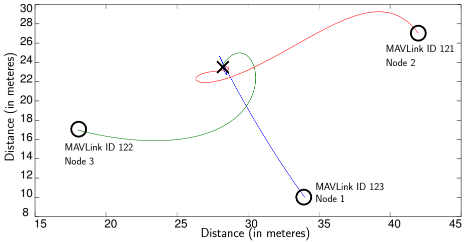
Simulation

- Quadrotor dynamics modeled as double-integrator dynamics decoupled along two principle axes
- Proposed consensus law first tested on a multi-agent system simulation platform with double integrator agents to predict the behaviour of the actual quadrotor system



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Hardware

Component	Details
Frame	Hobbyking X550
Motors	Turnigy Park 480 Outrunner 1320 kv BLDC
Motor driver	AE-30A Brushless ESC
Processor	APM 2.6 with ATmega 2560 microcontroller
Barometer	MS5611
IMU	MPU 6000 Gyroscope + Accelerometer
GPS	3DR uBlox GPS with Compass
Communication	XBee Pro S1, XBee S8
Power	LiPo 5000 mAh, 3S 50C, 11.1V battery

Table: List of hardware components used

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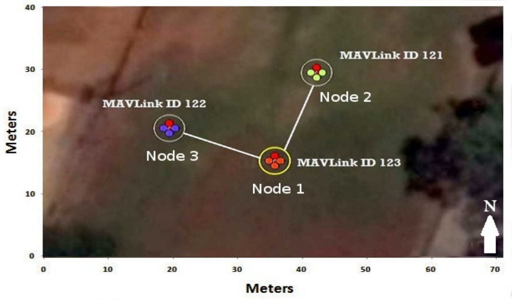
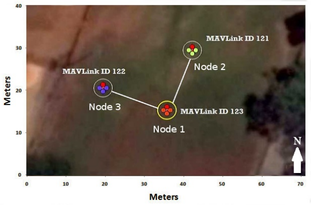
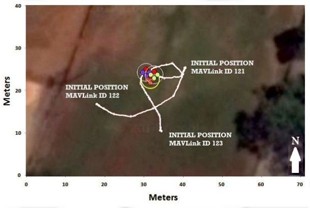


- Each quadrotor broadcasts its position information, $\mathbf{p}^E = \begin{bmatrix} p_x^E & p_y^E \end{bmatrix}^T$ which is measured by the on-board GPS receiver
- Position accuracy of the GPS receiver is 2.5m CEP
- Thus, for practical reasons, we say that the quadrotors reach consensus if $\|\mathbf{p}_i^E(t) - \mathbf{p}_j^E(t)\| \leq 2.5m$ for all $i, j = 1, 2, 3$.

Results

Consensus Video





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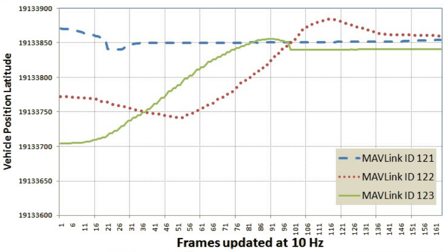
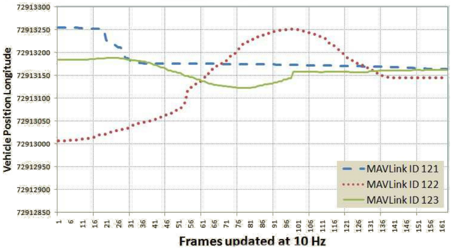
Position plots

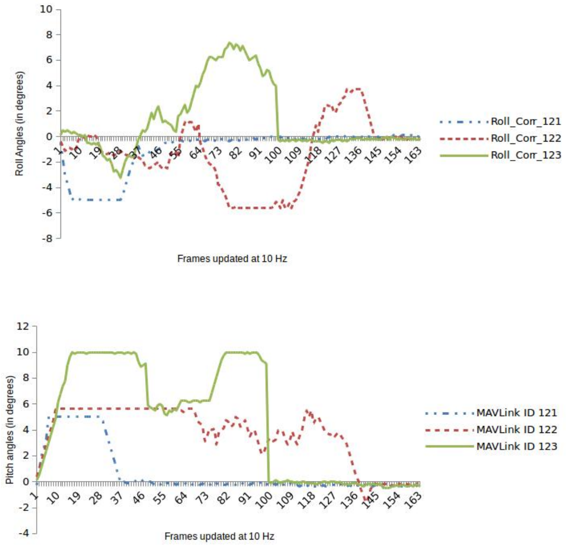
Latitude and Longitude tracking





Angle correction plots

Roll and pitch angle correction



Conclusion

Proposed and implemented a decentralized consensus law wherein

- On-board controllers take navigation decisions by communication with its neighbours $\implies$ decentralized!
- Justification for approximating the quadrotor as two independent double integrators acting along the x and $y-$ axes of motion
- Outdoor environment $\implies$ inherent GPS errors. However, the quadrotors still successfully managed to reach consensus.

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Thank you :)

Questions?

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